Topic 13

Integration

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

ALWAYS LEARNING

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 30

Q1

Find $\int (12x^5 - 8x^3 + 3)\,dx$, giving each term in its simplest form.

(Total for Question 1 is 4 marks)

Q2

Find

$$\int (12x^5 - 3x^2 + 4x^{\frac{1}{3}})\,dx$$

giving each term in its simplest form.

(Total for Question 2 is 5 marks)

Q3

Find

$$\int (6x^2 + \frac{2}{x^2} + 5)\,dx$$

giving each term in its simplest form.

(Total for Question 3 is 4 marks)

Q4

A curve with equation $y = f(x)$ passes through the point (2, 10). Given that

$$f'(x) = 3x^2 - 3x + 5$$

find the value of f(1).

(Total for Question 4 is 5 marks)

Q5

Use calculus to find the exact value of $\int_1^2 \left(3x^2 + 5 + \frac{4}{x^2}\right) \mathrm{d}x$

(Total for Question 5 is 5 marks)

Q6

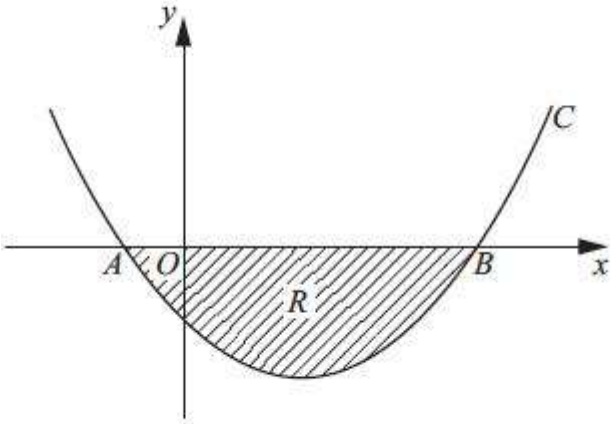

Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x + 1)(x - 5)$$

The curve crosses the x-axis at the points A and B.

(a) Write down the x-coordinates of A and B.

(1)

The finite region R, shown shaded in Figure 1, is bounded by C and the x-axis.

(b) Use integration to find the area of R.

(6)

(Total for Question 7 is 7 marks)

Bronze Mark Scheme

Q1.

Question Number	Scheme	Marks
	$(I=)\frac{12}{6}x^6-\frac{8}{4}x^4+3x+c$ $=2x^6-2x^4+3x+c$	M1 A1A1A1 **[4]**
	M1 for an attempt to integrate $x^n \to x^{n+1}$ (i.e. ax^6 or ax^4 or ax, where a is any non-zero constant). Also, this M mark can be scored for just the $+c$ (seen at some stage), even if no other terms are correct. 1st A1 for $2x^6$ 2nd A1 for $-2x^4$ 3rd A1 for $3x+c$ (or $3x+k$, etc., any appropriate letter can be used as the constant) Allow $3x^1+c$, but <u>not</u> $\frac{3x^1}{1}+c$. Note that the A marks can be awarded at separate stages, e.g. $\frac{12}{6}x^6-2x^4+3x$ scores 2nd A1 $\frac{12}{6}x^6-2x^4+3x+c$ scores 3rd A1 $2x^6-2x^4+3x$ scores 1st A1 (even though the c has now been lost). Remember that all the A marks are dependent on the M mark. If applicable, isw (ignore subsequent working) after a correct answer is seen. Ignore wrong notation if the intention is clear, e.g. Answer $\int 2x^6-2x^4+3x+c\,dx$.	

Q2.

Question Number	Scheme	Marks
	$\left(\int=\right)\frac{12x^6}{6}, -\frac{3x^3}{3}, +\frac{4x^{\frac{4}{3}}}{\frac{4}{3}}, (+c)$	M1A1,A1,A1
	$=\underline{2x^6 - x^3 + 3x^{\frac{4}{3}} + c}$	A1
		5
	Notes	
	M1 for some attempt to integrate: $x^n \to x^{n+1}$ i.e ax^6 or ax^3 or $ax^{\frac{4}{3}}$ or $ax^{\frac{1}{3}}$, where a is a non zero constant 1st A1 for $\frac{12x^6}{6}$ or better 2nd A1 for $-\frac{3x^3}{3}$ or better 3rd A1 for $\frac{4x^{\frac{4}{3}}}{\frac{4}{3}}$ or better 4th A1 for each term correct and simplified and the $+c$ occurring in the final answer	

Q3.

Question Number	Scheme	Marks
	$\left\{\int\left(6x^2+\frac{2}{x^2}+5\right)dx\right\} = \frac{6x^3}{3}+\frac{2x^{-1}}{-1}+5x\,(+c)$	M1 A1
	$= 2x^3 - 2x^{-1};+5x+c$	A1; A1
		4
	Notes	
	M1: for some attempt to integrate a term in x: $x^n \to x^{n+1}$ So seeing either $6x^2 \to \pm\lambda x^3$ or $\frac{2}{x^2} \to \pm\mu x^{-1}$ or $5 \to 5x$ is M1. 1st A1: for a correct un-simplified x^3 or x^{-1} $\left(\text{or } \frac{1}{x}\right)$ term. 2nd A1: for both x^3 and x^{-1} terms correct and simplified on the same line. Ie. $2x^3-2x^{-1}$ or $2x^3-\frac{2}{x}$. 3rd A1: for $+5x+c$. Also allow $+5x^1+c$. This needs to be written on the same line. **Ignore the incorrect use of the integral sign in candidates' responses.** Note: If a candidate scores M1A1A1A1 and their answer is NOT ON THE SAME LINE then withhold the final accuracy mark.	

Q4.

Question	Scheme	Marks
	$[\mathrm{f}(x)=]\frac{3x^3}{3}-\frac{3x^2}{2}+5x[+c]$ <u>or</u> $\left\{x^3-\frac{3}{2}x^2+5x(+c)\right\}$	M1A1
	$10=8-6+10+c$	M1
	$c=-2$	A1
	$\mathrm{f}(1)=1-\frac{3}{2}+5\ \text{"}-2\text{"}=\underline{\frac{5}{2}}$ (o.e.)	A1ft (5)
		5 marks
	Notes	
	1st M1 for attempt to integrate $x^n \to x^{n+1}$ 1st A1 all correct, possibly unsimplified. Ignore $+c$ here. 2nd M1 for using $x=2$ <u>and</u> $\mathrm{f}(2)=10$ to form a linear equation in c. Allow sign errors. They should be substituting into a <u>changed</u> expression 2nd A1 for $c=-2$ 3rd A1ft for $\frac{9}{2}+c$ Follow through their <u>numerical</u> c ($\neq 0$) This mark is dependent on 1st M1 and 1st A1 only.	

Q5.

Scheme	Marks	
$\int \left(3x^2 + 5 + 4x^{-2}\right)\mathrm{d}x = \frac{3x^3}{3} + 5x + \frac{4x^{-1}}{-1} \quad \left(= x^3 + 5x - 4x^{-1}\right)$	M1 A1 A1	
$\left[x^3 + 5x - 4x^{-1}\right]_1^2 = (8 + 10 - 2) - (1 + 5 - 4), \quad = 14$	M1, A1	(5)
		5
Integration: Accept any correct version, simplified or not. All 3 terms correct: M1 A1 A1, Two terms correct: M1 A1 A0, One power correct: M1 A0 A0. The given function must be integrated to score M1, and not e.g. $3x^4 + 5x^2 + 4$. Limits: M1: Substituting 2 and 1 into a 'changed function' and subtracting, either way round.		

Q6.

Question Number	Scheme		Marks
(a)	Seeing -1 and 5. (See note below.)		B1 (1)
(b)	$(x+1)(x-5) = \underline{x^2-4x-5}$ or $\underline{x^2-5x+x-5}$		B1
	$\int (x^2-4x-5)\,dx = \frac{x^3}{3} - \frac{4x^2}{2} - 5x\ \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 at least two out of three terms correctly ft.	M1A1ft A1
	$\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x\right]_{-1}^{5} = (......) - (......)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \begin{array}{l} \left(\frac{125}{3} - \frac{100}{2} - 25\right) - \left(-\frac{1}{3} - 2 + 5\right) \\ = \left(-\frac{100}{3}\right) - \left(\frac{8}{3}\right) = -36 \end{array} \right\}$		
	Hence, Area $= 36$	Final answer must be 36, not -36	A1 (6) [7]

	Notes
(a)	B1: for -1 and 5. Note that $(-1, 0)$ and $(5, 0)$ are acceptable for B1. Also allow $(0, -1)$ and $(0, 5)$ generously for B1. Note that if a candidate writes down that $A: (5,0)$, $B: (-1,0)$, (ie A and B interchanged,) then B0. Also allow values inserted in the correct position on the x-axis of the graph.
(b)	B1 for x^2-4x-5 or $x^2-5x+x-5$. If you believe that the candidate is applying the Way 2 method then $-x^2+4x+5$ or $-x^2+5x-x+5$ would then be fine for B1. 1st M1 for an attempt to integrate meaning that $x^n \to x^{n+1}$ for at least one of the terms. Note that $-5 \to 5x$ is sufficient for M1. 1st A1 at least two out of three terms correctly ft from their multiplied out brackets. 2nd A1 for correct integration only and no follow through. Ignore the use of a '$+c$'. Allow 2nd A1 also for $\frac{x^3}{3} - \frac{5x^2}{2} + \frac{x^2}{2} - 5x$. Note that $-\frac{5x^2}{2} + \frac{x^2}{2}$ only counts as one integrated term for the 1st A1 mark. Do not allow any extra terms for the 2nd A1 mark. 2nd M1: Note that this method mark is dependent upon the award of the first M1 mark in part (b). Substitutes 5 and -1 (and not 1 if the candidate has stated $x = -1$ in part (a).) (or the limits the candidate has found from part(a)) into an "integrated function" and subtracts, either way round. 3rd A1: For a final answer of 36, not -36. **Note:** An alternative method exists where the candidate states from the outset that Area $(R) = -\int_{-1}^{5}(x^2-4x+5)\,dx$ is detailed in the Appendix.

Question Number	Scheme		Marks
Aliter (b) Way 2	$(x+1)(x-5) = \underline{x^2-4x-5}$ or $\underline{x^2-5x+x-5}$	Can be implied by later working.	B1
	$-\int (x^2-4x-5)\,dx = -\frac{x^3}{3} + \frac{4x^2}{2} + 5x\ \{+c\}$	M: $x^n \to x^{n+1}$ for any one term. 1st A1 any two out of three terms correctly ft.	M1A1ft A1
	$\left[-\frac{x^3}{3} + \frac{4x^2}{2} + 5x\right]_{-1}^{5} = (......) - (......)$	Substitutes 5 and -1 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	$\left\{ \begin{array}{l} \left(-\frac{125}{3} + \frac{100}{2} + 25\right) - \left(\frac{1}{3} + 2 - 5\right) \\ = \left(\frac{100}{3}\right) - \left(-\frac{8}{3}\right) \end{array} \right\}$		
	Hence, Area $= 36$		A1 (6)

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Silver Questions

Calculators may not be used

The total mark for this section is 37

Q1

A curve has equation $y = \mathrm{f}(x)$ and passes through the point (4, 22).

Given that

$$\mathrm{f}'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $\mathrm{f}(x)$, giving each term in its simplest form.

(Total for Question 1 is 5 marks)

Q2

The gradient of a curve C is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(x^2+3)^2}{x^2}, \quad x \neq 0$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 6 + 9x^{-2}$

(2)

The point (3, 20) lies on C.

(b) Find an equation for the curve C in the form $y = \mathrm{f}(x)$.

(6)

(Total for Question 2 is 8 marks)

Q3

A curve has equation $y = \mathrm{f}(x)$. The point P with coordinates (9, 0) lies on the curve.

Given that

$$\mathrm{f}'(x) = \frac{x+9}{\sqrt{x}} \quad x > 0$$

(a) find $\mathrm{f}(x)$.

(6)

(b) Find the x-coordinates of the two points on $y = \mathrm{f}(x)$ where the gradient of the curve is equal to 10

(4)

(Total for Question 3 is 10 marks)

Q4

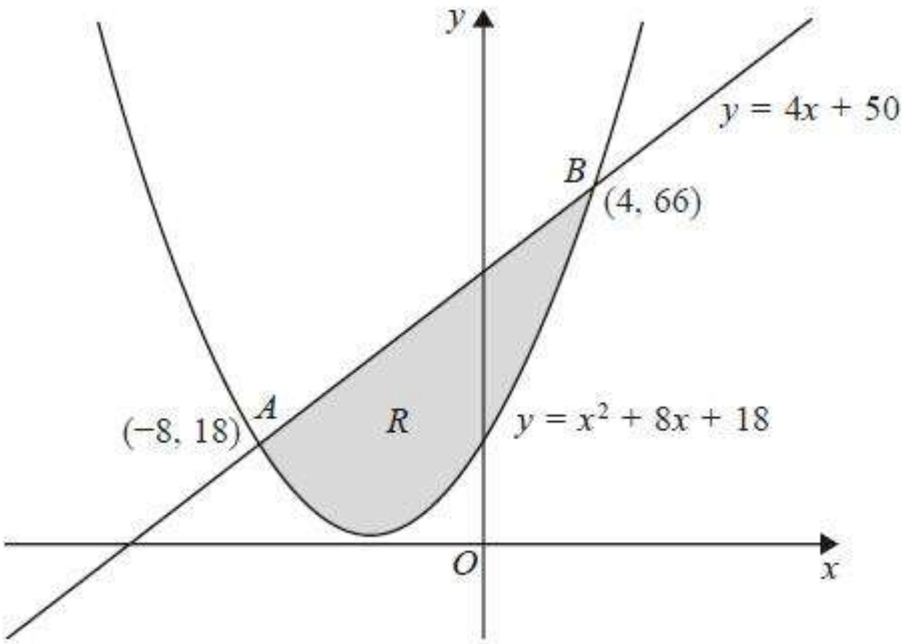

Figure 2

Figure 2 shows the line with equation $y = 4x + 50$ and the curve with equation y $= x^2 + 8x + 18$. The line cuts the curve at the points A (−8, 18) and B (4, 66).

The shaded region R is bounded by the line and the curve, as shown in Figure 2.

Using calculus, find the area of R.

(Total for Question 4 is 6 marks)

Q5

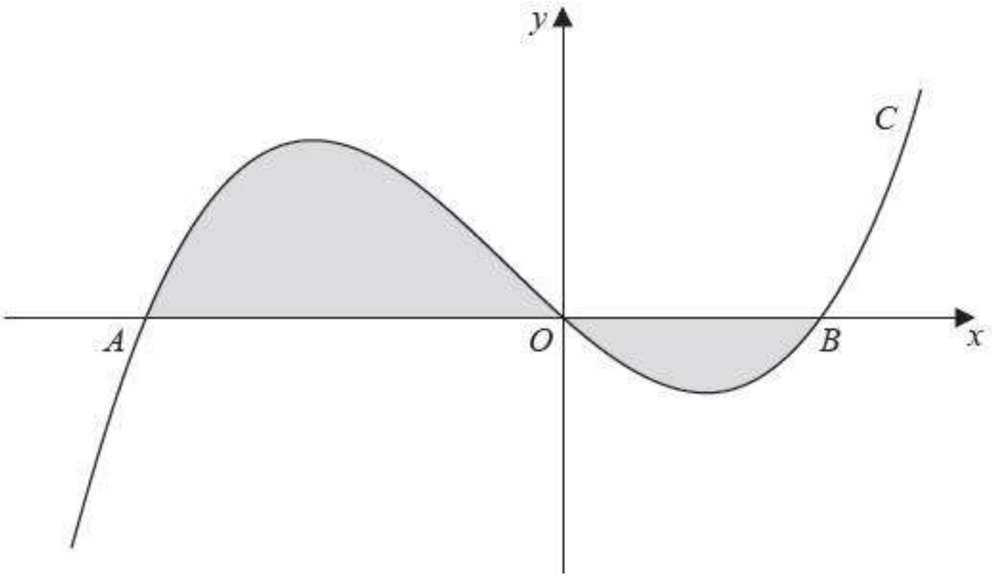

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = x(x + 4)(x - 2)$$

The curve C crosses the x-axis at the origin O and at the points A and B.

(a) Write down the x-coordinates of the points A and B.

(1)

The finite region, shown shaded in Figure 3, is bounded by the curve C and the x-axis.

(b) Use integration to find the total area of the finite region shown shaded in Figure 3.

(7)

(Total for Question 5 is 8 marks)

Silver Mark Scheme

Q1.

Question Number	Scheme	Marks
	$(\text{f}(x)=)\frac{3x^3}{3}-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-7x(+c)$	M1
	$=x^3-2x^{\frac{3}{2}}-7x\quad(+c)$	A1A1
	$\text{f}(4)=22\ \Rightarrow\ 22=64-16-28+c$	M1
	$c=2$	A1cso (5)
		[5]
	1st M1 for an attempt to integrate (x^3 or $x^{\frac{3}{2}}$ seen). The x term is insufficient for this mark and similarly the $+c$ is insufficient. 1st A1 for $\frac{3}{3}x^3$ or $-\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}$ (An unsimplified or simplified correct form) 2nd A1 for all three x terms correct and simplified… (the simplification may be seen later). The $+c$ is not required for this mark. Allow $-7x^1$, but <u>not</u> $-\frac{7x^1}{1}$. 2nd M1 for an attempt to use $x=4$ <u>and</u> $y=22$ in a changed function (even if differentiated) to form an equation in c. 3rd A1 for $c=2$ with no earlier incorrect work (a final expression for f(x) is not required).	

Q2.

Question Number	Scheme	Marks
(a)	$(x^2+3)^2=x^4+3x^2+3x^2+3^2$	M1
	$\frac{(x^2+3)^2}{x^2}=\frac{x^4+6x^2+9}{x^2}=x^2+6+9x^{-2}\quad(*)$	A1 cso (2)
(b)	$y=\frac{x^3}{3}+6x+\frac{9}{-1}x^{-1}(+c)$	M1 A1 A1
	$20=\frac{27}{3}+6\times3-\frac{9}{3}+c$	M1
	$c=-4$	A1
	$[y=]\frac{x^3}{3}+6x-9x^{-1}-4$	A1 ft (6)
		(8 marks)

Q3.

Question Number	Scheme	Notes	Marks
(a)	$f'(x)=\dfrac{x+9}{\sqrt{x}}=\dfrac{x}{\sqrt{x}}+\dfrac{9}{\sqrt{x}}=x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$	M1: Correct attempt to split into 2 separate terms or fractions. May be implied by one correct term. Divides by $x^{\frac{1}{2}}$ or multiplies by $x^{-\frac{1}{2}}$. A1: $x^{\frac{1}{2}}+9x^{-\frac{1}{2}}$ or equivalent	M1A1
	$f(x)=\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}+9\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}(+c)$	M1: Independent method mark for $x^n\to x^{n+1}$ on separate terms A1: Allow un-simplified answers. No requirement for + c here	M1A1
	$\dfrac{(9)^{\frac{3}{2}}}{\frac{3}{2}}+9\dfrac{(9)^{\frac{1}{2}}}{\frac{1}{2}}+c=0\Rightarrow c=\ldots$	Substitutes $x=9$ and $y=0$ into their integrated expression leading to a value for c. If no c at this stage M0A0 follows unless their method implies that they are correctly finding a constant of integration.	M1
	$f(x)=\dfrac{2}{3}x^{\frac{3}{2}}+18x^{\frac{1}{2}}-72$	There is no requirement to simplify their f(x) so accept any correct un-simplified form.	A1
			(6)
(b)	$f'(x)=\dfrac{x+9}{\sqrt{x}}=10\Rightarrow x+9=10\sqrt{x}$	Sets $f'(x)=\dfrac{x+9}{\sqrt{x}}=10$ and multiplies by $\sqrt{x}$. The terms in x must be in the numerator. E.g. allow $\dfrac{x+9}{10}=\sqrt{x}$	M1
	They must be setting either the original $f'(x)=10$ or an equivalent <u>correct</u> expression = 10		
	$(\sqrt{x}-9)(\sqrt{x}-1)=0\Rightarrow\sqrt{x}=\ldots$	Correct attempt to solve a relevant 3TQ in $\sqrt{x}$ leading to solution for $\sqrt{x}$. **Dependent on the previous M1.**	dM1
	$x=81, x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a <u>correct</u> equation.	A1, B1
			(4)
			[10]
Alternative to part (b)	$\left(\dfrac{x+9}{\sqrt{x}}\right)^2=10^2\Rightarrow x^2+18x+81=100x$	Sets $\dfrac{x+9}{\sqrt{x}}=10$, squares and multiplies by x. They must be setting either the original $f'(x)=10$ or an equivalent <u>correct</u> expression = 10	M1
	$(x-81)(x-1)=0\Rightarrow x=..$	Correct attempt to solve a relevant 3TQ leading to solution for x. **Dependent on the previous M1.**	dM1
	$x=81, x=1$	Note that the $x=1$ solution could be just written down and is B1but must come from a <u>correct</u> equation.	A1, B1

Q4.

Question Number	Scheme	Marks
	Way 1: Area of trapezium $= \frac{1}{2}(a+b)\times h = \frac{1}{2}(18+66)\times "(4--8)" =$ or may use combination of rectangle and triangle to find trapezium area or may use integration $\int_{-8}^{4}(4x+50)\mathrm{d}x = \left[2x^2+50x\right]_{-8}^{4} = (232)-(128-400) =$	M1
	504 (may be implied by correct final answer)	A1
	$\int x^2+8x+18\mathrm{d}x = \frac{1}{3}x^3+4x^2+18x$	M1 A1
	Use limits 4 **and** -8 $[(\frac{1}{3}(4)^3+4(4)^2+18\times 4)-(\frac{1}{3}(-8)^3+4.(-8)^2+18\times(-8))] = A_1\ (=216)$ **And** uses correct combination of correct areas. Area of region = Area of trapezium $-\ A_1$	dM1
	$=504-\left(\frac{472}{3}--\frac{176}{3}\right) = 288$	A1 [6]
	Way 2: Alternative method using "line – curve" **Sets up** $y=4x+50-(x^2+8x+18)$	M1 A1
	$\int -x^2-4x+32\mathrm{d}x = -\frac{x^3}{3}-2x^2+32x$	M1A1
	Use limits 4 and -8 on this *subtracted* integration	dM1
	Obtains 288	A1 [6]

Q5.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td>Seeing -4 and 2.</td><td>B1
(1)</td></tr>
<tr><td>(b)</td><td>$x(x+4)(x-2) = \underline{x^3+2x^2-8x}$ or $\underline{x^3-2x^2+4x^2-8x}$ (without simplifying)</td><td>B1</td></tr>
<tr><td></td><td>$\int(x^3+2x^2-8x)\,dx = \frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\{+c\}$ or $\frac{x^4}{4}-\frac{2x^3}{3}+\frac{4x^3}{3}-\frac{8x^2}{2}\{+c\}$</td><td>M1A1ft</td></tr>
<tr><td></td><td>$\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{0} = (0)-\left(64-\frac{128}{3}-64\right)$ or $\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{0}^{2} = \left(4+\frac{16}{3}-16\right)-(0)$</td><td>dM1</td></tr>
<tr><td></td><td>One integral $= \pm 42\frac{2}{3}$ (42.6 or awrt 42.7) or other integral $= \pm 6\frac{2}{3}$ (6.6 or awrt 6.7)</td><td>A1</td></tr>
<tr><td></td><td>Hence Area $=$ "$their42\frac{2}{3}$"+"$their6\frac{2}{3}$" or Area $=$ "$their42\frac{2}{3}$"$-$"$-their6\frac{2}{3}$"</td><td>dM1</td></tr>
<tr><td></td><td>$= 49\frac{1}{3}$ or 49.3 or $\frac{148}{3}$ (NOT $-\frac{148}{3}$)</td><td>A1</td></tr>
<tr><td></td><td>(An answer of $= 49\frac{1}{3}$ may not get the final two marks – check solution carefully)</td><td>(7)
[8]</td></tr>
<tr><td></td><td>Notes for Question</td><td></td></tr>
<tr><td>(a)</td><td>B1: Need both -4 and 2. May see (-4,0) and (2,0) (correct) but allow (0,-4) and (0, 2) or $A = -4$, $B = 2$ or indeed any indication of -4 and 2 – check graph also</td><td></td></tr>
<tr><td>(b)</td><td>B1: Multiplies out cubic correctly (terms may not be collected, but if they are, mark collected terms here)
M1: Tries to integrate their expansion with $x^n \to x^{n+1}$ for at least one of the terms
A1ft: completely correct integral following through from their CUBIC expansion (if only quadratic or quartic this is A0)
dM1: (dependent on previous M) substituting EITHER $-a$ and 0 and subtracting either way round OR similarly for 0 and b. If their limits $-a$ and b are used in ONE integral, apply the Special Case below.
A1: Obtain either $\pm 42\frac{2}{3}$ (or 42.6 or awrt 42.7)from the integral from -4 to 0 or $\pm 6\frac{2}{3}$ (6.6 or awrt 6.7) from the integral from 0 to 2; NO follow through on their cubic (allow decimal or improper equivalents $\frac{128}{3}$ or $\frac{20}{3}$) isw such as subtracting from rectangles. This will be penalized in the next two marks, which will be M0A0.
dM1 (depends on first method mark) Correct method to obtain shaded area so adds two positive numbers (areas) together or uses their positive value minus their negative value, obtained from two separate definite integrals.
A1: Allow 49.3, 49.33, 49.333 etc. Must follow correct logical work with no errors seen.
For full marks on this question there must be two definite integrals, from -4 to 0 and from 0 to 2, though the evaluations for 0 may not be seen.
(Trapezium rule gets no marks after first two B marks)</td><td></td></tr>
<tr><td>(b)</td><td>Special Case: one integral only from $-a$ to b: B1M1A1 available as before, then
$\left[\frac{x^4}{4}+\frac{2x^3}{3}-\frac{8x^2}{2}\right]_{-4}^{2} = (4+\frac{16}{3}-16)-\left(64-\frac{128}{3}-64\right) = -6\frac{2}{3}+42\frac{2}{3} = \ldots$ dM1 for correct use of their limits $-a$ and b and subtracting either way round.
A1 for 36: NO follow through. Final M and A marks not available. Max 5/7 for part (b)</td><td></td></tr>
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Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

Given that $\dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and the value of q

(2)

Given that $\dfrac{dy}{dx} = \dfrac{6x + 3x^{\frac{5}{2}}}{\sqrt{x}}$ and that $y = 90$ when $x = 4$,

(b) find y in terms of x, simplifying the coefficient of each term.

(5)

(Total for Question 1 is 7 marks)

Q2

$$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x} \text{ where } x > 0$$

Given that $y = 37$ at $x = 4$, find y in terms of x, giving each term in its simplest form.

(Total for Question 2 is 7 marks)

Q3

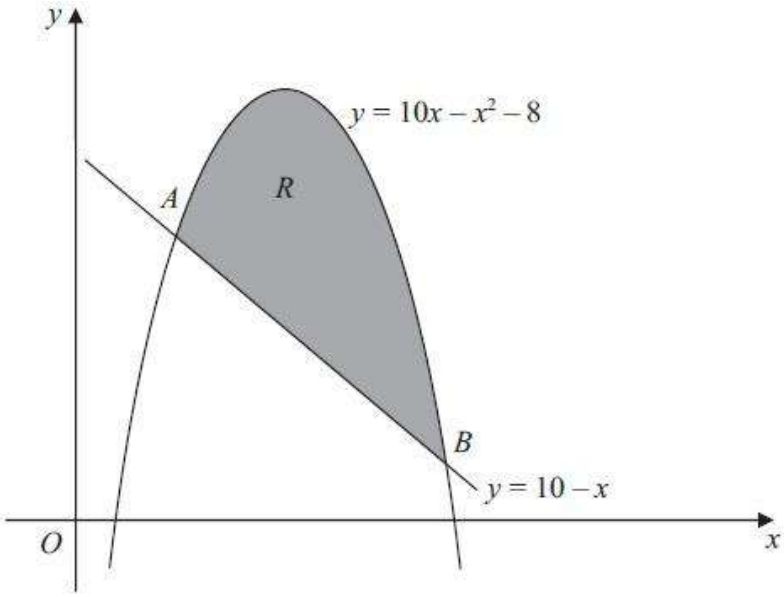


Figure 2

Figure 2 shows the line with equation $y = 10 - x$ and the curve with equation $y = 10x - x^2 - 8$
The line and the curve intersect at the points A and B, and O is the origin.

(a) Calculate the coordinates of A and the coordinates of B.

(5)

The shaded area R is bounded by the line and the curve, as shown in Figure 2.

(b) Calculate the exact area of R.

(7)

(Total for Question 3 is 12 marks)

Q4

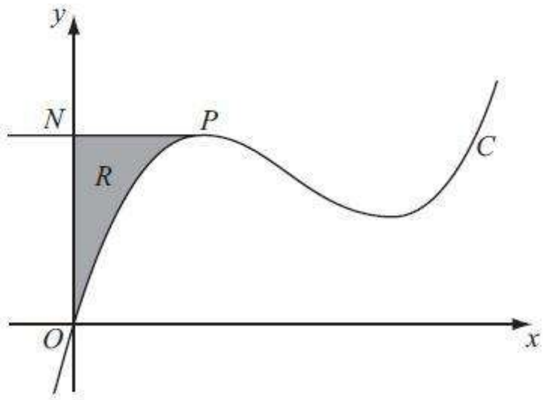


Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = x^3 - 10x^2 + kx \ ,$$

where k is a constant.

The point P on C is the maximum turning point.

Given that the x-coordinate of P is 2,

(a) show that $k = 28$.

(3)

The line through P parallel to the x-axis cuts the y-axis at the point N.
The region R is bounded by C, the y-axis and PN, as shown shaded in Figure 2.

(b) Use calculus to find the exact area of R.

(6)

(Total for Question 4 is 9 marks)

Gold Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$p = \frac{1}{2}, q = 2$ or $6x^{\frac{1}{2}}, 3x^2$	B1, B1 (2)
(b)	$\frac{6x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{3x^3}{3} \quad \left(= 4x^{\frac{3}{2}} + x^3\right)$ $x = 4, y = 90:\ 32 + 64 + C = 90 \Rightarrow C = -6$ $y = 4x^{\frac{3}{2}} + x^3 + \text{"}their - 6\text{"}$	M1 A1ft M1 A1 A1 (5) 7
	Notes	
	(a) Accept any equivalent answers, e.g. $p = 0.5$, $q = 4/2$ (b) 1st M: Attempt to integrate $x^n \to x^{n+1}$ (for either term) 1st A: ft their p and q, but terms need not be simplified ($+C$ not required for this mark) 2nd M: Using $x = 4$ and $y = 90$ to form an equation in C. 2nd A: cao 3rd A: answer as shown with simplified correct coefficients and powers – but follow through their value for C If there is a 'restart' in part (b) it can be marked independently of part (a), but marks for part (a) cannot be scored for work seen in (b). Numerator and denominator integrated separately: First M mark **cannot** be awarded so only mark available is second M mark. So 1 out of 5 marks.	

Q2.

Question Number	Scheme	Marks
	$\frac{dy}{dx} = 6x^{-\frac{1}{2}} + x\sqrt{x}$	
	$x\sqrt{x} = x^{\frac{3}{2}}$	B1
	$x^n \to x^{n+1}$	M1
	$y = \frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}(+c)$	A1, A1
	Use $x = 4, y = 37$ to give equation in c, $37 = 12\sqrt{4} + \frac{2}{5}(\sqrt{4})^5 + c$	M1
	$\Rightarrow c = \frac{1}{5}$ or equivalent eg. 0.2	A1
	$(y) = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$	A1
		(7 marks)

B1 $x\sqrt{x} = x^{\frac{3}{2}}$. This may be implied by $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ oe in the subsequent work.

M1 $x^n \to x^{n+1}$ in at least one case so see either $x^{\frac{1}{2}}$ or $x^{\frac{5}{2}}$ or both

A1 One term integrated correctly. It does not have to be simplified Eg. $\frac{6}{\frac{1}{2}}x^{\frac{1}{2}}$ or $+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}$.

No need for $+c$

A1 Other term integrated correctly. See above. No need to simplify nor for $+c$. Need to see

$\frac{6}{\frac{1}{2}}x^{\frac{1}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}}$ or a simplified correct version

M1 Substitute $x = 4, y = 37$ to produce an equation in c.

A1 Correctly calculates $c = \frac{1}{5}$ or equivalent e.g. 0.2

A1 cso $y = 12x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + \frac{1}{5}$. Allow $5y = 60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1$ and accept fully simplified equivalents.

e.g. $y = \frac{1}{5}(60x^{\frac{1}{2}} + 2x^{\frac{5}{2}} + 1)$, $y = 12\sqrt{x} + \frac{2}{5}\sqrt{x^5} + \frac{1}{5}$

Q3.

Question number	Scheme		Marks
Method 1 (a)	Puts $10-x=10x-x^2-8$ and rearranges to give three term quadratic	Or puts $y=10(10-y)-(10-y)^2-8$ and rearranges to give three term quadratic	M1
	Solves their "$x^2-11x+18=0$" using acceptable method as in general principles to give $x=$	Solves their "$y^2-9y+8=0$" using acceptable method as in general principles to give $y=$	M1
	Obtains $x=2$, $x=9$ (may be on diagram or in part (b) in limits)	Obtains $y=8$, $y=1$ (may be on diagram)	A1
	Substitutes their x into a given equation to give $y=$ (may be on diagram)	Substitutes their y into a given equation to give $x=$ (may be on diagram or in part (b))	M1
	$y=8$, $y=1$	$x=2$, $x=9$	A1 (5)
(b)	$\int(10x-x^2-8)\,dx = \frac{10x^2}{2}-\frac{x^3}{3}-8x\,\{+c\}$		M1 A1 A1
	$\left[\frac{10x^2}{2}-\frac{x^3}{3}-8x\right]_2^9 = (\ldots\ldots)-(\ldots\ldots)$ $=90-\frac{4}{3}=88\frac{2}{3}$ or $\frac{266}{3}$		dM1
	Area of trapezium $=\frac{1}{2}(8+1)(9-2)=31.5$		B1
	So area of R is $88\frac{2}{3}-31.5=57\frac{1}{6}$ or $\frac{343}{6}$		M1A1 cao (7)
			12 marks
Notes (a)	First **M1**: See scheme Second **M1**: See notes relating to solving quadratics Third **M1**: This may be awarded if one substitution is made Two correct Answers following tables of values, or from Graphical calculator are 5/5 **Just one pair of correct coordinates – no working or from table is M0M0A0M1A0**		
(b)	**M1**: $x^n \to x^{n+1}$ for any one term. **1st A1**: at least two out of three terms correct **2nd A1**: All three correct **dM1**: Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round (NB: If candidate **changes all signs** to get $\int(-10x+x^2+8)\,dx = -\frac{10x^2}{2}+\frac{x^3}{3}+8x\,\{+c\}$ This is M1 A1 A1 Then uses limits dM1 and trapezium is B1 Needs to *change sign of value obtained* from integration for final M1A1 so $-88\frac{2}{3}-31.5$ is M0A0) **B1**: Obtains 31.5 for area under line using any correct method (could be integration) or triangle minus triangle $\frac{1}{2}\times8\times8-\frac{1}{2}$ or rectangle plus triangle [may be implied by correct 57 1/6] **M1**: Their Area under curve – Their Area under line (if integrate both need same limits) **A1**: Accept 57.16recurring but not 57.16 **PTO for Alternative method**		

Method 2 for (b)	Area of R $= \int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ 3rd M1 (in (b)): Uses difference between two functions in integral.	
	$\int_2^9 -x^2 + 11x - 18\,dx$ M: $x^n \to x^{n+1}$ for any one term.	M1
	$= -\frac{x^3}{3} + \frac{11x^2}{2} - 18x \{+c\}$ A1 at least two out of these three simplified terms	A1
	Correct integration. (Ignore + c).	A1
	$\left[-\frac{x^3}{3} + \frac{11x^2}{2} - 18x\right]_2^9 = (\ldots\ldots) - (\ldots\ldots)$ Substitutes 9 and 2 (or limits from part(a)) into an "integrated function" and subtracts, either way round.	dM1
	This mark is implied by final answer which rounds to 57.2	B1
	See above working(allow bracketing errors) to decide to award 3rd M1 mark for (b) here:	M1
	$40.5 - (-16\tfrac{2}{3}) = 57\tfrac{1}{6}$ cao	A1
		(7)
Special case of above method	$\int_2^9 x^2 - 11x + 18\,dx = \frac{x^3}{3} - \frac{11x^2}{2} + 18x \{+c\}$	M1A1A1
	$\left[\frac{x^3}{3} - \frac{11x^2}{2} + 18x\right]_2^9 = (\ldots\ldots) - (\ldots\ldots)$	DM1
	This mark is implied by final answer which rounds to 57.2 (not -57.2)	B1
	Difference of functions implied (see above expression)	M1
	$40.5 - (-16\tfrac{2}{3}) = 57\tfrac{1}{6}$ cao	A1
		(7)
Special Case 2	Integrates expression in y e.g. "$y^2 - 9y + 8 = 0$": This can have first M1 in part (b) and no other marks. (It is not a method for finding this area)	
Notes	Take away trapezium again having used Method 2 loses last two marks Common Error: Integrates $-x^2 + 9x - 18$ is likely to be M1A1A0dM1B0M1A0 Integrates $2 - 11x - x^2$ is likely to e M1A0A0dM1B0M1A0 Writing $\int_2^9 (10x - x^2 - 8) - (10 - x)\,dx$ only earns final M mark	

Q4.

Question Number	Scheme		Marks
	(a) $\frac{dy}{dx} = 3x^2 - 20x + k$	(Differentiation is required)	M1 A1
	At $x = 2$, $\frac{dy}{dx} = 0$, so $12 - 40 + k = 0$ $\quad k = 28$	(*)	A1 cso
	N.B. The '= 0' must be seen at some stage to score the final mark.		
	Alternatively: (using $k = 28$)		
	$\frac{dy}{dx} = 3x^2 - 20x + 28$	(M1 A1)	
	'Assuming' $k = 28$ only scores the final cso mark if there is justification that $\frac{dy}{dx} = 0$ at $x = 2$ represents the maximum turning point.		(3)
	(b) $\int (x^3 - 10x^2 + 28x)\,dx = \frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}$	Allow $\frac{kx^2}{2}$ for $\frac{28x^2}{2}$	M1 A1
	$\left[\frac{x^4}{4} - \frac{10x^3}{3} + 14x^2\right]_0^2 = \ldots$	$\left(= 4 - \frac{80}{3} + 56 = \frac{100}{3}\right)$	M1
	(With limits 0 to 2, substitute the limit 2 into a 'changed function')		
	y-coordinate of $P = 8 - 40 + 56 = 24$	Allow if seen in part (a)	B1
	(The B1 for 24 may be scored by implication from later working)		
	Area of rectangle $= 2 \times$ (their y-coordinate of P)		
	Area of $R =$ (their 48) $- \left(\text{their } \frac{100}{3}\right) = \frac{44}{3}$ $\left(14\frac{2}{3} \text{ or } 14.\dot{6}\right)$		M1 A1
	If the subtraction is the 'wrong way round', the final A mark is lost.		(6) 9

	(a) M: $x^n \to cx^{n-1}$ (c constant, $c \neq 0$) for one term, seen in part (a).	
	(b) 1st M: $x^n \to cx^{n+1}$ (c constant, $c \neq 0$) for one term. Integrating the gradient function loses this M mark.	
	2ndM: Requires use of limits 0 and 2, with 2 substituted into a 'changed function'. (It may, for example, have been differentiated).	
	Final M: Subtract their values either way round. This mark is dependent on the use of calculus and a correct method attempt for the area of the rectangle.	
	A1: Must be exact, not 14.67 or similar, but isw after seeing, say, $\frac{44}{3}$.	
	Alternative: (effectively finding area of rectangle by integration) $\int \{24 - (x^3 - 10x^2 + 28x)\}\,dx = 24x - \left(\frac{x^4}{4} - \frac{10x^3}{3} + \frac{28x^2}{2}\right)$, etc.	
	This can be marked equivalently, with the 1st A being for integrating the same 3 terms correctly. The 3rd M (for subtraction) will be scored at the same stage as the 2nd M. If the subtraction is the 'wrong way round', the final A mark is lost.	

Platinum Questions

Calculators may not be used

The total mark for this section is 10

1 (a) On the same diagram, sketch $y = x$ and $y = \sqrt{x}$, for $x \geq 0$, and mark clearly the coordinates of the points of intersection of the two graphs.

(2)

(b) With reference to your sketch, explain why there exists a value a of x ($a > 1$) such that

$$\int_0^a x \, dx = \int_0^a \sqrt{x} \, dx.$$

(2)

(c) Find the exact value of a.

(4)

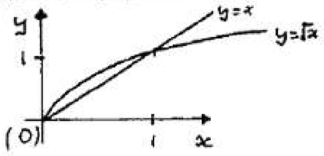
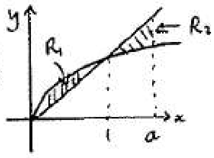
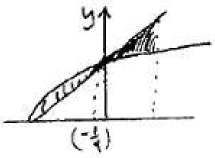
(d) Hence, or otherwise, find a non-constant function $f(x)$ and a constant b ($b \neq 0$) such that

$$\int_{-b}^{b} f(x) \, dx = \int_{-b}^{b} \sqrt{[f(x)]} \, dx.$$

(2)

(Total for Question 1 is 10 marks)

Platinum Mark Scheme

Working		Notes	Marks	
2. (a) 		Relative shapes	B1	
		0 or (0,0) implied and (1,1). On axes is OK.	B1	(2)
(b) $\left[R_1 = \frac{1}{6}\right]$ 	As a increases from 1 R_2 increases. Choose a so that $R_2 = R_1$ then areas are the same.	Diagram with regions or mention of areas.	B1g	
		Full argument	B1h	(2)
(c) $\int_0^a x\,dx = \int_0^a x^{\frac{1}{2}}dx \Rightarrow \left[\frac{x^2}{2}\right]_0^a = \left[\frac{2}{3}x^{3/2}\right]_0^a$		Attempt both integrals – one correct	M1	
$\Rightarrow \frac{a^2}{2} = \frac{2}{3}a^{3/2}$		A correct equation in a	A1	
$\Rightarrow a^{3/2}(3a^{\frac{1}{2}} - 4) = 0$	$\rightarrow a^{1/2} = 4/3$ o.e.	Attempt to solve $\rightarrow a^{\wedge} = k$	M1	
$a = \underline{\underline{\frac{16}{9}}}$			A1	(4)
(d) 	Translate $\frac{1}{2}a$ ←.			
	$f(x) = \underline{\underline{x + \frac{8}{9}}}$	$x + \frac{a}{2} = f(x)$ (Any suitable $f(x) \pm b$)	B1ft	
	$\underline{\underline{b = \frac{8}{9}}}$	$\frac{a}{2} = b$ ft their a.	B1ft	(2)
				10

S.C. if $b = \beta$ and $f(x) = x + \beta$ score B1 only